Nonsymmetrized Correlations in Quantum Noninvasive Measurements

Adam Bednorz,[1] Christoph Bruder,[2] Bertrand Reulet,[3] and Wolfgang Belzig[4]

[1]*Faculty of Physics, University of Warsaw ul. Hoża 69, PL00-681 Warsaw, Poland*
[2]*Department of Physics, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland*
[3]*Département de Physique, Université de Sherbrooke, Sherbrooke, QC, J1K2R1,Canada*
[4]*Fachbereich Physik, Universität Konstanz, D-78457 Konstanz, Germany*

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A long-standing problem in quantum mesoscopic physics is how to order operators in quantum noise expressions like $\langle I(-\omega)I(\omega)\rangle$, where $I(\omega)$ is the measured current at frequency ω. Symmetrized ordering of operators describes a classical measurement while nonsymmetrized ordering corresponds to a quantum detector, sensitive to either emission or absorption of photons. We show that both ordering schemes can be embedded in quantum weak-measurement theory taking into account a finite interaction time between the system and the detector. Importantly, the nonsymmetrized ordering reveals its nonclassical nature already in second-order correlations, contrary to symmetrized ordering. This feature can be related to the squeezing of the many-body state of the transported electrons in a ac-driven tunnel junction.

Quantum measurement theory has been based on the projection postulate [1], although nowadays it includes generalized schemes based on auxiliary detectors, described mathematically by positive operator-valued measures (POVM) [2]. To specify a POVM requires arguments based on physical considerations such as detector efficiency, or the assumption of thermal equilibrium. A real physical interaction generally leads to backaction on the system to be measured, which makes the interpretation of measurements difficult. Hence, all detection schemes are in general invasive as the measured system is perturbed. The disturbance is strongest for projective measurements, as the information in the measurement basis is completely erased. In contrast, other POVM schemes can be much less disturbing, as is often the case in experiments [3–5].

To avoid invasiveness, Aharonov, Albert and Vaidman [6] studied the limit of weak measurement, in which the system is coupled so weakly to the detector that it remains almost untouched. The price to pay is a large detection noise, which is however completely independent of the system. The gain is that other measurements on a non-compatible observable can be performed. After the subtraction of the detector noise, the statistics of the measurements has a well-defined limit for vanishing coupling, which for incompatible observables turns out to be described by a quasiprobability and not a real probability distribution [7, 8].

The most common weak-measurement theories assume that the system-detector interaction is instantaneous [9–14]. Such a Markovian measurement scheme justifies the symmetrized ordering of operators, relevant for many experiments [3]: $\langle I(-\omega)I(\omega)\rangle \to \langle \hat{I}(-\omega)\hat{I}(\omega) + \hat{I}(\omega)\hat{I}(-\omega)\rangle/2$ (denoting quantum expectation values $\langle\hat{X}\rangle = \mathrm{Tr}\hat{X}\hat{\rho}$ for an initial state $\hat{\rho}$). Here, $\hat{I}(\omega) = \int dt\ \ \hat{I}(t)e^{i\omega t}$ is the Fourier transform of the time-dependent current $\hat{I}(t)$ in the Heisenberg picture. However, certain experiments, e. g. involving absorptive detectors, are well described by nonsymmetrized correlators like $\langle\hat{I}(-\omega)\hat{I}(\omega)\rangle$, corresponding to emission noise for $\omega \geq 0$ [4, 15–20]. These experiments clearly lie beyond the scope of Markovian weak measurement theory.

In this Letter, we formulate a general theory of weak detection which allows for the description of nonsymmetrized correlators. Whereas no Markovian scheme of weak measurement predicts emission noise, we show that the latter appears naturally if one allows for a measurement with *memory*. In fact, non-Markovian weak measurements follow just from a few natural assumptions imposed on the POVM in the limit of weak coupling. By further requiring that no information transfer occurs in thermal equilibrium we fix uniquely the scheme, which contains the detector temperature as only parameter. Our scheme interpolates between emission and absorption measurements, when varying the detector temperature. As it is independent of other properties of the detector, our scheme applies to circuit QED, mesoscopic current measurements and quantum optical systems equally well. Interestingly, applied to a simple harmonic oscillator, the scheme reveals classical equations of motion but with the initial distribution corresponding to the Glauber-Sudarshan P-function [21]. Contrary to the instantaneous measurement, the non-Markovian scheme can violate weak positivity [19]. To test it, we propose a measurement of photon-assisted current-fluctuations, which are shown to violate a Cauchy-Schwarz type inequality. Identifying the finite-frequency current operators with quadratures in analogy to quantum optics, we show that the thus created non-equilibrium state of the current is squeezed and therefore has essentially nonclassical correlations.

We start by developing a the general framework of weak quantum measurement based upon the POVM formalism including non-Markovian features. We consider a set of n *independent* detectors continuously recording n time-dependent signals $a_j(t)$ for $j = 1,\dots,n$. Each

detector is related to an observable $\hat{A}_j$. For example, n ammeters are inserted in a complex circuit: $a_j(t)$ is the recorded current in the branch j and $\hat{I}_j(t)$ the current operator in that branch. Note that in general $\hat{A}_j(t)$ and $\hat{A}_k(t')$ do not commute even if $j \neq k$ since $\hat{A}_j$ and $\hat{A}_k$ may not commute with the hamiltonian. We want to relate classical correlators of measured quantities like $\langle a_1(t_1)\cdots a_n(t_n)\rangle$ to their equivalent for weak quantum measurements $\langle\cdots\rangle_w$. These should involve *linear* correlators of the $\hat{A}_j$, which can be taken at different times to allow for memory effects of the detectors, while preserving *causality*. The requirements of linearity and causality are fulfilled by replacing $a_j(t)$ in the correlator by a superoperator $\int dt' \check{A}_j^{t-t'}(t')$ and perform time ordering, i.e.:

$$\langle a_1(t_1)\cdots a_n(t_n)\rangle_w = \mathrm{Tr}\int \mathrm{d}^n t'\, \mathcal{T}\left[\check{A}_n^{t_n-t'_n}(t'_n)\cdots\check{A}_1^{t_1-t'_1}(t'_1)\right]\hat{\rho} \qquad (1)$$

Here $\mathcal{T}$ denotes time ordering with respect to the arguments in brackets, $\hat{\rho}$ is the density matrix, and $\check{A}$ are superoperators defined as:

$$\check{A}_j^{t-t'}(t') = g_j(t-t')\check{A}_j^c(t') + f_j(t-t')\check{A}_j^q(t')/2\,. \qquad (2)$$

The superoperators $\check{A}_j^{c/q}$ [22] act on any operator $\hat{X}$ like an anticommutator/commutator: $\check{A}_j^c\hat{X} = \{\hat{A}_j,\hat{X}\}/2$ and $\check{A}_j^q\hat{X} = i[\hat{A}_j,\hat{X}]$. In the above expressions we supposed for simplicity that the detectors are in a stationary state so that only time differences $t_j - t'_j$ matter. It is easy to show that the correlator (1) is invariant if we change the order of the a_j, and that all combinations of A_j that preserve causality can be reproduced.

We will also assume that the average of single measurements coincides with the usual average for projective measurements, i.e. $\langle a_j(t)\rangle_w = \langle\hat{A}_j(t)\rangle$. This implies $g_j(t-t') = \delta(t-t')$. Other choices of g simply mimic the effect of classical frequency filters. Thus the only freedom left is the choice of the real function f_j that multiplies $\check{A}_j^q$. Note that $f_j(t)$ can be non-zero for $t>0$ without violating causality, since it is accompanied by $\check{A}_j^q$ and only future measurements are affected. For the last measurement, future effects disappear because the leftmost $\check{A}^q$ vanishes under the trace in Eq. (1). For simplicity, we will assume a single $f = f_j$, independent of j. The limit $f = 0$ corresponds to the Markovian case.

Now we want to show that correlations obeying these requirements can be obtained from the general quantum measurement formalism. Based on Kraus operators $\hat{K}$ [23], the probability distribution of the measurement results is $\rho = \langle\check{K}\rangle$ for $\check{K}\hat{X} = \hat{K}\hat{X}\hat{K}^\dagger$, where the only condition on $\hat{K}$ is that the outcome probability is normalized regardless of the input state $\hat{\rho}$. Here we need $\hat{K}$ to be time-dependent. In general, we assume that $\hat{K}[\hat{A},a]$ is a functional of the whole time history of observables $\hat{A}(t)$ and outcomes $a(t)$. We shall assume that the functional $\hat{K}$ is stationary so it depends only on relative time arguments.

The essential step to satisfy Eq. (1) is to take the limit $\hat{K}\sim\hat{1}$ which corresponds to a noninvasive measurement. This can be obtained from an arbitrary initial POVM by rescaling $\hat{K}[\hat{A},a]\to\hat{K}_\eta = C(\eta)\hat{K}[\eta\hat{A},\eta a]$ with $\eta\to 0$, which defines $\rho_\eta = \langle\check{K}_\eta\rangle$. Here $C(\eta)$ is a normalization factor.

The desired correlation function (1) can be derived by the special limiting procedure for an almost general POVM, namely

$$\langle a_1(t_1)\cdots a_n(t_n)\rangle_w = \lim_{\eta\to 0}\langle a_1(t_1)\cdots a_n(t_n)\rangle_\eta \qquad (3)$$

where the average on the right-hand side is with respect to ρ_η. We assume the absence of internal correlations between different detectors, namely $\hat{K}[\hat{A},a] = \mathcal{T}\prod_j \hat{K}[\hat{A}_j,a_j]$, where $\mathcal{T}$ applies to the time arguments of $\hat{A}$.

Expanding

$$\hat{K}[\hat{A},a]/k[a] = 1 + \int \mathrm{d}t' F[a,t']\hat{A}(t') + \mathcal{O}(\hat{A}^2)\,, \qquad (4)$$

we find

$$\check{K}/|k[a]|^2 = 1 + \int \mathrm{d}t'\,\left(2\mathrm{Re}F[a,t']\check{A}^c(t') + \mathrm{Im}F[a,t']\check{A}^q(t')\right) + \mathcal{O}(A^2)\,. \qquad (5)$$

Here, $k[a]$ is a functional amplitude independent of the properties of the system which represents the detection noise. As we want the measurement to be noninvasive to lowest order, we impose the condition that $\int F[a,t']|k[a]|^2\mathrm{D}a$ vanishes; $\mathrm{D}a$ is the functional measure. Our conditions imply that $\int 2a(t)\mathrm{Re}F[a,t']|k[a]|^2\mathrm{D}a = \delta(t-t')$, and we get $f(t-t') = \int 2a(t)\mathrm{Im}F[a,t']|k[a]|^2\mathrm{D}a$. Thus, the most general weak Kraus operator takes the form given in Eq. (5), which is our main result. A particular Gaussian example of a POVM realizing this scheme is presented in the Supplementary Material A.

To discuss the consequences of different forms of f, we now calculate the noise spectral density,

$$S_{ab}(\omega) = \int \mathrm{d}t\; e^{i\omega t}\langle a(t)b(0)\rangle_w\,. \qquad (6)$$

An important special case is a system in a thermal equilibrium state, $\hat{\rho}\sim\exp(-\hat{H}/k_BT)$. We further assume that the averages of $\hat{A}$ and $\hat{B}$ vanish. If the detector temperature T_d is equal to T and in the absence of other nonequilibrium effects (like a bias voltage, or special initial conditions), we expect that no information transfer from the system to the detector occurs, i.e., that $S_{ab}(\omega) = 0$. This requirement leads to a necessary

condition on the form of f (see Supplementary Material B): $f(\omega) = -i(2n_B(\omega) - 1) = -i\coth(\hbar\omega/2k_BT_d)$, where $n_B(\omega)$ is the Bose distribution at temperature T_d. Equivalently, $f(t) = -k_BT_d\coth(\pi t k_BT_d/\hbar)/\hbar$ (at zero temperature $f(t) = -1/\pi t$). We use the name *equilibrium ordering* for this special choice of f. In a recent work, the zero temperature case has been called *time-normal* [24].

The necessary form of f is also sufficient. Indeed, the property $S_{ab}(\omega) = 0$ follows from the fluctuation-dissipation theorem [25]

$$\int \mathrm{d}t\, e^{i\omega t}\langle\hat{A}(t)\hat{B}(0)\rangle_T = \int \mathrm{d}t\, e^{i\omega t + \hbar\omega/k_BT}\langle\hat{B}(0)\hat{A}(t)\rangle_T\,, \tag{7}$$

because for an arbitrary stationary state we get

$$S_{ab}(\omega) = \int \mathrm{d}t\, e^{i\omega t}\langle e^{\hbar\omega/2k_BT_d}\hat{B}(0)\hat{A}(t) \\ -e^{-\hbar\omega/2k_BT_d}\hat{A}(t)\hat{B}(0)\rangle/\sinh(\hbar\omega/2k_BT_d)\,, \tag{8}$$

which reduces to the emission noise [18]

$$\int e^{i\omega t}\mathrm{d}t\,\langle\theta(-\omega)\hat{A}(t)\hat{B}(0) + \theta(\omega)\hat{B}(0)\hat{A}(t)\rangle \tag{9}$$

for zero detector temperature $T_d = 0$. Thus, for $T_d \neq T$, $S_{ab}(\omega)$ is in general not equal to zero and contains information about the system. Reversing the sign of f will reverse ω in (9), transforming it into absorption noise. Equilibrium ordering is relevant for experimental situations like in [4] and consistent with the quantum tape [19] or photodetection model [20] if the temperature of the tape (or the photons) is T_d.

It is interesting to note that for this special choice of f the higher-order fluctuations also vanish if $\hat{\rho} \propto \exp(-\hat{H}/k_BT)$ and $T = T_d$. We can write the Fourier transform of (1) as

$$\left\langle\prod_k a_k(\omega_k)\right\rangle_w = \int \mathrm{d}^n t\, e^{i\sum_k \omega_k t_k} \\ \times\mathrm{Tr}\mathcal{T}\prod_k\sum_\pm \frac{\pm e^{\pm\hbar\omega_k/2k_BT}\check{A}_k^\pm(t_k)}{2\sinh(\hbar\omega_k/2k_BT)}\hat{\rho}\,, \tag{10}$$

with $\check{A}^+\hat{X} = \hat{A}\hat{X}$ and $\check{A}^-\hat{X} = \hat{X}\hat{A}$. Now, we can split $\hat{\rho} = \hat{\rho}^{1/2}\hat{\rho}^{1/2}$, expand the above expression as a sum of operator products and move one factor $\hat{\rho}^{1/2}$ leftwards and the other rightwards so that they meet again at the trace sign, which gives (1) in the form

$$\int \mathrm{d}^n t\, e^{i\sum_k \omega_k t_k}\mathrm{Tr}\hat{\rho}\mathcal{T}\prod_k \\ \sum_\pm \frac{\pm e^{\pm\hbar\omega_k/2k_BT}\check{A}_k^\pm(t_k \mp i\hbar/2k_BT)}{2\sinh(\hbar\omega_k/2k_BT)}\,. \tag{11}$$

Shifting $t \to t \pm i\hbar/2k_BT$ and using $\mathrm{Tr}\check{A}^q\ldots = 0$ leads to

$$\int \mathrm{d}^n t\, e^{\sum_k i\omega_k t_k}\mathrm{Tr}\hat{\rho}\mathcal{T}\prod_k \check{A}_k^q(t_k)/2i\sinh(\hbar\omega_k/2k_BT) = 0. \tag{12}$$

The proof is hence completed for all nonzero frequencies. In every physical situation it extends also to zero frequency because the cumulants are continuous functions of frequency.

Surprisingly, the equilibrium ordering differs qualitatively from the symmetrized one, when one considers weak positivity, i.e., the classical nature of second-order correlations [8, 26]. The symmetrized correlation matrix $C_{ab} = \langle ab\rangle = \langle\hat{A}\hat{B} + \hat{B}\hat{A}\rangle/2$ is positive definite so the Gaussian probability distribution $\propto \exp(-\sum_{ab} C^{-1}_{ab}ab/2)$ reproduces all first and second order symmetrized quantum correlations. This is not the case in equilibrium ordering. If $T_d > 0$, then already for a system in equilibrium at $T = 0$ and $\omega > 0$ we get $\langle a(-\omega)a(\omega)\rangle = -\langle\hat{A}(\omega)\hat{A}(-\omega)\rangle e^{-\hbar\omega/2k_BT_d}/2\sinh(\hbar\omega/2k_BT)$, which is negative. For $T_d = 0$ and stationary states again the weak positivity holds, because the correlation matrix (9) is positive definite. However, it is violated for nonstationary states. This can be demonstrated using a two level system with the Hamiltonian $\hat{H} = \hbar\Omega\hat{\sigma}_z/2$, with observable $\hat{A} = \hat{\sigma}_x + \hat{\sigma}_z$ and the initial state $\hat{\rho}(0) = (\hat{1} + \hat{\sigma}_y)/2$. By direct calculation (Supplementary Material C) we find $\langle a^2(0)\rangle_w = -(2/\pi)\ln\Omega t_\infty$, where t_∞ is a cutoff set by intrinsic decoherence or detector backaction. Thus, weak positivity is obviously violated. In reality $\langle a^2(0)\rangle_w$ corresponds to $\langle a(0)b(0)\rangle$ where the detectors a and b monitor the same observable $\hat{A} = \hat{B}$ in (1).

There is an interesting connection between equilibrium ordering and the Glauber-Sudarshan P function [21]. Let us take the harmonic oscillator $\hat{H} = \hbar\Omega\left(\hat{p}^2 + \hat{x}^2\right)/2$, with $[\hat{x},\hat{p}] = i$, and consider correlations with respect to the quasiprobability (1) with antisymmetric $f(t)$. In this case the time ordering is irrelevant as shown in Supplementary Material D. Let us define $\check{A}(t) = \check{A}^c(t) + \int dt'\, f(t-t')\check{A}^q(t')/2$. The evolution of $\check{x}(t)$ and $\check{p}(t)$ is just classical,

$$\check{x}(t) = \check{x}\cos(\Omega t) + \check{p}\sin(\Omega t), \\ \check{p}(t) = \check{p}\cos(\Omega t) - \check{x}\sin(\Omega t) \tag{13}$$

while for $t = 0$ (since $f(t)$ is antisymmetric and real) we have

$$\check{x} = \check{x}^c + i\check{p}^q f(\Omega)/2, \\ \check{p} = \check{p}^c - i\check{x}^q f(\Omega)/2\,. \tag{14}$$

We now define the ladder operators through $\hat{a}^\dagger = (\hat{x} + i\hat{p})/\sqrt{2}$ with $[\hat{a},\hat{a}^\dagger] = \hat{1}$. This leads to $\check{a} = \check{a}^c - f(\Omega)\check{a}^q/2$ and $\check{a}^\dagger = \check{a}^{\dagger c} + f(\Omega)\check{a}^{\dagger q}/2$. In the zero-temperature case, $f(\Omega) = -i$, and defining $\alpha = (x + ip)/\sqrt{2}$ we get the

quasiprobabilistic average $\langle\alpha^n\alpha^{*k}\rangle = \mathrm{Tr}\hat{a}^n\hat{\rho}\hat{a}^{\dagger k}$. On the other hand, this is a property of the Glauber-Sudarshan function $P(\alpha)$, defined by $\hat{\rho} = \int \mathrm{d}^2\alpha\ P(\alpha)|\alpha\rangle\langle\alpha|$ for normalized coherent states $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$, $\langle\alpha|\alpha\rangle = 1$ [21]. Since $\langle\alpha^n\alpha^{*k}\rangle_P = \int \mathrm{d}^2\alpha\ \alpha^n\alpha^{*k}P(\alpha) = \mathrm{Tr}\hat{a}^n\hat{\rho}\hat{a}^{\dagger k}$, we find that the initial quasiprobability for a zero-temperature detector is identical to $P(\alpha, t = 0)$. It is interesting to note that reversing the sign of f leads to the Husimi-Kano Q function instead of P [2], while $f = 0$ gives the Wigner function [7, 27].

The fact that we obtain the P-function shows the deep connection between the non-Markovian weak measurement formalism and the quantum-optical detector theory. One of the interesting consequences is that zero-temperature equilibrium ordering is consistent with photoabsorptive detection schemes, in which the P-function appears naturally [2]. It is also interesting to draw a link between the violation of weak positivity in equilibrium ordering and the properties of squeezed states. The ground state of a harmonic oscillator fulfills $\langle\hat{x}^2\rangle = 1/2$, which corresponds to $\langle x^2\rangle_P = 0$. A squeezed state can be such that $\langle\hat{x}^2\rangle < 1/2$, still minimizing the Heisenberg uncertainty principle. This translates into a negative variance of the position described by the (quasiprobability) P-function, i.e. $\langle x^2\rangle_P < 0$ [28] and is therefore equivalent to a violation of weak positivity.

Let us now consider how our results apply to the case of current fluctuations in mesoscopic conductors The quantum description of the noise in the junction, $S_I(\omega) = \int \mathrm{d}t\ e^{i\omega t}\langle\delta I(t)\delta I(0)\rangle$, where $\delta I(t) = I(t) - \langle I(t)\rangle$, will depend on the choice of f in (1). For $f = 0$, we get symmetrized noise $S_I^s = G\hbar\sum_\pm w(\omega \pm eV/\hbar)/2$, where G is the conductance, V is the constant bias voltage and $w(\alpha) = \alpha\coth(\hbar\alpha/k_BT)$ [19]. For a choice of f with an arbitrary T_d, we obtain $S_I = S_I^s - G\hbar w_d(\omega)$. Hence, the detection schemes differ by a term that is independent of the voltage and the temperature of the system, making it impossible to detect non-classicality in this scheme.

An experimentally feasible test of squeezing and violation of weak positivity is possible using a coherent conductor (e.g. a tunnel junction for the sake of simplicity) subject to an AC voltage bias $V(t) = V_{ac}\cos\Omega t$. Consider the classical inequality

$$|\delta I(\omega) - \delta I(-\omega)|^2 \geq 0 \Rightarrow \langle|\delta I(\omega)|^2\rangle \geq \mathrm{Re}\langle\delta I^2(\omega)\rangle\,. \quad (15)$$

For symmetrized ordering one gets [29]

$$\langle\{\delta\hat{I}(\omega), \delta\hat{I}(\omega')\}\rangle/2 = 2\pi\hbar G\sum_m \delta(\omega + \omega' - 2m\Omega)$$
$$\sum_n J_n(eV_{ac}/\hbar\Omega)J_{n-2m}(eV_{ac}/\hbar\Omega)w(\omega - n\Omega)\,, \quad (16)$$

where J_n are the Bessel functions. In the case of equilibrium ordering at $T_d = 0$ one only has to subtract $2\pi\hbar G|\omega|\delta(\omega + \omega')$ from the above result. As shown in Fig. 1, the classical inequality is violated for $\omega = \Omega$ in

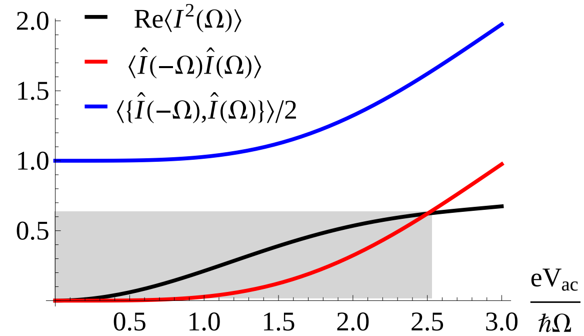

FIG. 1: (color online) The plot shows the quantum correlation functions in tunnel junctions normalized by $2\pi G\hbar\Omega t_0$ at $T = 0$. The emission noise, at $T_d = 0$, (red line) violates the classical inequality (15) for a certain range of $eV_{ac}/\hbar\Omega$ (shaded region). The violation is equivalent to the squeezing condition (17) for the symmetrized noise (blue line).

a certain range of $eV_{ac}/\hbar\Omega$, but only in equilibrium ordering. This can be reinterpreted in terms of the existence of squeezing in the quantum shot noise : consider the two quadratures associated with the finite-frequency current operator: $\hat{A} = i[\delta\hat{I}(\omega) - \delta\hat{I}(-\omega)]/2$ and $\hat{B} = [\delta\hat{I}(\omega) + \delta\hat{I}(-\omega)]/2$. Using $\langle[\hat{I}(\omega), \hat{I}(-\omega)]\rangle = 2t_0G\hbar\omega$, we find [30] $\langle[\hat{A}, \hat{B}]\rangle = it_0G\hbar\omega$ (with the total detection time t_0). Thus the squeezing condition [2]

$$\langle\hat{A}^2\rangle < |\langle[\hat{A}, \hat{B}]\rangle|/2 \quad (17)$$

is related to the violation of weak positivity, $\langle A^2\rangle_w < 0$ in equilibrium ordering (1) with $T_d = 0$ and allows to violate Eq. (15). Hence, according to Fig. 1, quantum shot noise with ac-driving creates current states, which resemble squeezed light for a certain range of the ac-voltage.

In conclusion, we have presented a theory of a generic weak-measurement scheme that includes emission noise. It requires a non-Markovian POVM with a specially chosen memory function f, which has no analog in the Markovian picture. The scheme is consistent with the expectation about the absence of information flow between system and detector in equilibrium at a given temperature. Hence any detection requires a nonequilibrium situation. Another direct consequence is that even the simple Markovian detection process must involve a nonequilibrium detector state. Finally, nonsymmetrized ordering leads to a violation of weak positivity, which can be tested experimentally by violation of suitable inequalities, equivalent to the squeezing condition in some cases.

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lence Research Chairs program (BR).

[1] J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, 1932).

[2] H. M. Wiseman and G. J. Milburn, *Quantum Measurement and Control* (Cambridge University Press, Cambridge, 2009).

[3] E. Zakka-Bajjani *et al.*, Phys. Rev. Lett. **99**, 236803 (2007); J. Gabelli and B. Reulet, Phys. Rev. Lett. **100**, 026601 (2008); J. Stat. Mech. P01049 (2009); A. Palacios-Laloy *et al.*, Nat. Phys. **6**, 442 (2010).

[4] R. Deblock *et al.*, Science 301, 203 (2003); A. H. Safavi-Naeini *et al.*, Phys. Rev. Lett. **108**, 033602 (2012).

[5] N. Katz *et al.*, Phys. Rev. Lett. **101**, 200401 (2008); D. Bozyigit *et al.*, Nature Phys. **7**, 154 (2011).

[6] Y. Aharonov, D.Z. Albert, and L. Vaidman, Phys. Rev. Lett. **60**, 1351 (1988).

[7] A. Bednorz and W. Belzig, Phys. Rev. Lett. **105**, 106803 (2010).

[8] A. Bednorz, W. Belzig, and A. Nitzan, New J. Phys. **14**, 013009 (2012).

[9] N. S. Williams and A. N. Jordan, Phys. Rev. Lett. **100**, 026804 (2008); A. Romito, Y. Gefen, and Y. M. Blanter, Phys. Rev. Lett. **100**, 056801 (2008).

[10] J. Audretsch and M. Mensky, Phys. Rev. A **56**, 44 (1997).

[11] V. Chernyak, F. Sanda, and S. Mukamel, Phys. Rev. E **73**, 036119 (2006).

[12] M. Tsang, Phys. Rev. A **80**, 033840 (2009).

[13] B. Berg, L. I. Plimak, A. Polkovnikov, M. K. Olsen, M. Fleischhauer, and W. P. Schleich, Phys. Rev. A **80**, 033624 (2009).

[14] J. Dressel, S. Agarwal, and A.N. Jordan, Phys. Rev. Lett. **104**, 240401 (2010).

[15] G.B. Lesovik and R. Loosen, JETP Lett. **65**, 295 (1997).

[16] R. Aguado and L.P. Kouwenhoven, Phys. Rev. Lett. **84**, 1986 (2000).

[17] U. Gavish, Y. Levinson, and Y. Imry, Phys. Rev. B **62**, R10637 (2000).

[18] A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, Rev. Mod. Phys. **82**, 1155 (2010).

[19] A. Bednorz and W. Belzig, Phys. Rev. B **81**, 125112 (2010).

[20] C.W.J. Beenakker and H. Schomerus, Phys. Rev. Lett. **86**, 700 (2001); **93**, 096801 (2004); A.V. Lebedev, G.B. Lesovik, and G. Blatter, Phys. Rev. B **81**, 155421 (2010).

[21] E. C. G. Sudarshan, Phys. Rev. Lett. **10**, 277 (1963); R. J. Glauber, Phys. Rev. **131**, 2766 (1963).

[22] R. Zwanzig, Physica **30**, 1109 (1964).

[23] K. Kraus, *States, Effects and Operations* (Springer, Berlin, 1983).

[24] L.I. Plimak, S.T. Stenholm, and W.P. Schleich, Physica Scr. **T147**, 014026 (2012).

[25] H.B. Callen and T.A. Welton, Phys. Rev. **83**, 34 (1951).

[26] A. Bednorz and W. Belzig, Phys. Rev. B **83**, 125304 (2011).

[27] A. Bednorz and W. Belzig, Phys. Rev. A **83**, 052113 (2011).

[28] E. Shchukin, Th. Richter, and W. Vogel, Phys. Rev. A **71**, 011802(R) (2005).

[29] G. B. Lesovik and L. S. Levitov, Phys. Rev. Lett. **72**, 538 (1994); J. Gabelli and B. Reulet, arXiv:0801.1432; A. Crepieux, P. Devillard, and T. Martin, Phys. Rev. B **69**, 205302 (2004); D. Chevallier *et al.*, Phys. Rev. B **81**, 205411 (2010).

[30] U. Gavish, B. Yurke and Y. Imry, Phys. Rev. Lett. **93**, 250601 (2004).

Supplementary Material

A. GAUSSIAN NON-MARKOVIAN POVM

An example of a POVM leading to a non-Markovian weak measurement is the Gaussian Kraus functional

$$\hat{K}[\hat{A},a]\propto \mathcal{T}\exp\int \mathrm{d}t\left[-(\hat{A}(t)-a(t))^2+\int \mathrm{d}t'\; if(t-t')(2a(t)-\hat{A}(t)\theta(t-t'))\hat{A}(t')\right]. \tag{A.1}$$

Here, the first term in the exponent is the Markovian part, while the second term describes the non-Markovian measurement process including a fixed but arbitrary real function $f(t)$, characterizing the memory effect. The Heaviside function θ is necessary to ensure the normalization of the Kraus operator. Following the standard procedure we find the Kraus superoperator in the form

$$\check{K}[\hat{A},a]\propto \tag{A.2}$$
$$\mathcal{T}\exp\int \mathrm{d}t\left[-2(\check{A}^c(t)-a(t))^2+(\check{A}^q(t))^2/2+\int \mathrm{d}t'\;2f(t-t')(a(t)\check{A}^q(t')-\theta(t-t')(\check{A}^c(t)\check{A}^q(t')+\check{A}^q(t)\check{A}^c(t'))\right].$$

To prove the normalization, $\int \mathrm{D}a\langle\check{K}\rangle=1$, we perform the Gaussian integral over a (time ordering is no problem if kept up throughout the calculation) and get

$$\int \mathrm{D}a\;\check{K}=\mathcal{T}\exp\left[\int \mathrm{d}t(\check{A}^q(t))^2/2+\int \mathrm{d}t'\theta(t'-t)2f(t-t')\check{A}^q(t')\check{A}^c(t)\mathrm{d}t'\right.$$
$$\left.-\theta(t-t')2f(t-t')\check{A}^q(t)\check{A}^c(t')\mathrm{d}t'+f(t-t')f(t-t'')\check{A}^q(t')\check{A}^q(t'')\mathrm{d}t'\mathrm{d}t''/2\right], \tag{A.3}$$

where we have ordered properly $\check{A}^q(t')$ and $\check{A}^c(t)$. In the power expansion, omitting the identity term, the leftmost superoperator is always $\check{A}^q$. Since $\mathrm{Tr}\check{A}^q\cdots=0$ we obtain $\int \mathrm{D}a\;\langle\check{K}[\hat{A},a]\rangle=1$ or $\int \mathrm{D}a\;\hat{K}^\dagger\hat{K}=\hat{1}$. In general, we define $\hat{K}[\hat{A},a]$ for n measurements as $\hat{K}[\hat{A},a]=\mathcal{T}\prod_j\hat{K}[\hat{A}_j,a_j]$. Note that putting $\hat{A}=0$ gives Gaussian white noise $\rho\propto e^{-2a^2}$, which leads to the large detection noise in the weak limit, $\rho_\eta\propto e^{-2\eta^2a^2}$, that has to be subtracted/deconvoluted from the experimental data.

B. FIXING THE MEMORY FUNCTION f

Since the detector function $f(\omega)$ should be system-independent in thermal equilibrium, any system can be used to determine it. We therefore consider a 2-level system with $\hat{A}=\hat{B}=\hat{\sigma}_x$ and $\hat{H}=\hbar\Omega\hat{\sigma}_z/2$. The requirement $S(\omega)=0$ is equivalent to

$$\mathrm{Re}\int_{-\infty}^{0}e^{i\omega t}\mathrm{d}t\;\langle(1-if(\omega))\hat{\sigma}_x(t)\hat{\sigma}_x(0)+(1+if(\omega))\hat{\sigma}_x(0)\hat{\sigma}_x(t)\rangle=0\,. \tag{B.1}$$

The equilibrium state reads $\hat{\rho}=(\hat{1}-\hat{\sigma}_z\tanh(\hbar\Omega/2k_BT))/2$ and

$$\langle\hat{\sigma}_x(0)\hat{\sigma}_x(t)\rangle=\langle\hat{\sigma}_x(-t)\hat{\sigma}_x(0)\rangle=\cos(\Omega t)+i\tanh(\hbar\Omega/2k_BT)\sin(\Omega t)\,, \tag{B.2}$$

and (B.1) leads to the requirement that

$$\mathrm{Re}\left[\frac{1}{\epsilon+i\omega+i\Omega}+\frac{1}{\epsilon+i\omega-i\Omega}+if(\omega)\left(\frac{1}{\epsilon+i\omega+i\Omega}-\frac{1}{\epsilon+i\omega-i\Omega}\right)\tanh\left(\frac{\hbar\Omega}{2k_BT}\right)\right] \tag{B.3}$$

vanishes for $\epsilon\to 0_+$. Since $\frac{1}{\epsilon+ix}=\frac{1}{ix}+\pi\delta(x)$, we can ignore the delta function for $\omega\neq\pm\Omega$, and the vanishing of (B.3) reduces to $\mathrm{Re}f(\omega)=0$. As f must be independent of the system, we are free to choose Ω, $\mathrm{Re}f(\omega)=0$ must hold for all ω, including $\omega=\pm\Omega$. So f is purely imaginary, and (B.3) reads

$$\delta(\omega+\Omega)+\delta(\omega-\Omega)-(\delta(\omega+\Omega)-\delta(\omega-\Omega))\tanh\left(\frac{\hbar\Omega}{2k_BT}\right)\mathrm{Im}f(\omega)\,. \tag{B.4}$$

Therefore, $1\pm\mathrm{Im}f(\omega)\tanh(\hbar\Omega/2k_BT)$ at $\omega=\pm\Omega$, and $f(\omega)=-i\coth(\hbar\omega/2k_BT)$.

C. VIOLATION OF WEAK POSITIVITY

From (1) we find for $f(t) = -1/\pi t$

$$\langle a(t)b(s)\rangle_w = \left\langle \{\hat{A}(t),\hat{B}(s)\}/2 + \int_{-\infty}^{s} i\mathrm{d}t'[\hat{A}(t'),\hat{B}(s)]/2\pi(t-t') + \int_{-\infty}^{t} i\mathrm{d}s'[\hat{B}(s'),\hat{A}(t)]/2\pi(s-s')\right\rangle . \tag{C.1}$$

For $\hat{H} = \hbar\Omega\sigma_z/2$ and $\hat{A} = \hat{B} = \hat{\sigma}_x + \hat{\sigma}_z$ we find $\hat{A}(t) = \hat{\sigma}_x \cos\Omega t - \hat{\sigma}_y \sin\Omega t + \hat{\sigma}_z$ and $i[\hat{A},\hat{A}(t)]/2 = \hat{\sigma}_x \sin\Omega t - \hat{\sigma}_z \sin\Omega t + \hat{\sigma}_y(\cos\Omega t - 1)$. Therefore, for $\hat{\rho}(0) = (\hat{1} + \hat{\sigma}_y)/2$ we get

$$\langle a^2\rangle = 2 + \frac{2}{\pi}\int_0^\infty \mathrm{d}t \frac{\cos\Omega t - 1}{t} . \tag{C.2}$$

For small t the integral is convergent but for large t only $\cos\Omega t/t$ converges. The remaining integral $\int \mathrm{d}t/t$ diverges logarithmically and one should put a cutoff at t_∞.

Certainly no experiment will record infinite correlations. The cutoff t_∞ is in practice bounded by the decoherence time of the system and the measurement noise (which also diverges). The infinity would occur only in the limit of zero measurement strength and a perfect two-level system, which is impossible.

D. HARMONIC OSCILLATOR

For the harmonic oscillator with $\hat{H} = \hat{p}^2/2m + m\omega^2\hat{x}^2$ and $[\hat{x},\hat{p}] = i\hbar$, we have $[\hat{x}(t),\hat{p}(t')] = i\hbar\hat{1}\cos(\Omega(t-t'))$, $[\hat{x}(t),\hat{x}(t')] = -i\hbar\hat{1}\sin(\Omega(t-t'))/m\Omega$, $[\hat{p}(t),\hat{p}(t')] = -i\hbar\hat{1}\sin(\Omega(t-t'))m\Omega$, so the commutator depends only on the difference $t-t'$, which applies also to superoperators. To see that the time ordering is irrelevant, let us take linear functions A,B of x and p and calculate

$$\begin{aligned}
&\left(\check{A}^c(t) + \int dt'\ f(t-t')\check{A}^q(t')/2\right)\left(\check{B}^c(s) + \int ds'\ f(s-s')\check{B}^q(s')/2\right)\\
&-\mathcal{T}\left(\check{A}^c(t) + \int dt'\ f(t-t')\check{A}^q(t')/2\right)\left(\check{B}^c(s) + \int ds'\ f(s-s')\check{B}^q(s')\right)\\
&= \int dt'\ f(t-t')\theta(s-t')[\check{B}^c(s),\check{A}^q(t')]/2 + \int ds'\ f(s-s')\theta(t-s')[\check{B}^q(t),\check{A}^c(s')]/2\\
&= \int du\ \theta(u)(f(u+t-s)[\check{B}^c(s),\check{A}^q(s-u)] + f(s-t-u)[\check{B}^q(t+u),\check{A}^c(t)])/2 .
\end{aligned} \tag{D.1}$$

The last expression vanishes because of the antisymmetry of f and the fact that the commutators depend only on the difference in time arguments. The proof generalize to multiple products because $[\check{B}^{c/q}(s),\check{A}^{c/q}(u)]$ is proportional to identity superoperator for $A,B = x,p$.